Fundamentals of Electronics Resit

Prof. dr. E. Chicca, dr. J.S. Blijleven

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Information

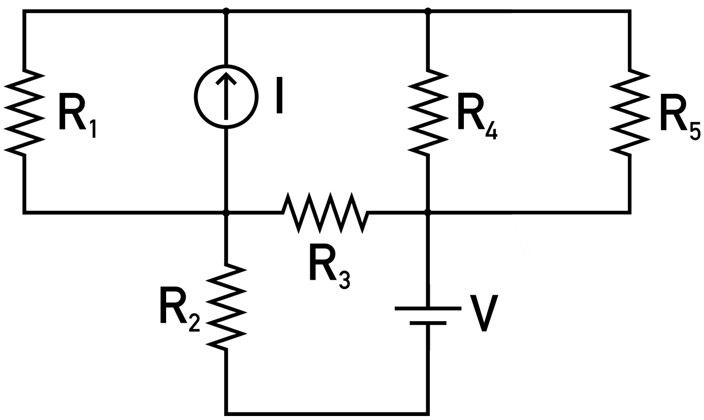
Welcome to the Fundamental of Electronics resit. Please read carefully the information below.

How to write your solution
Please use a **pen** and not a pencil. Make sure your **hand writing** is understandable by others. **Drawings** do not need to be beautiful/perfect but it is important that they are easy to understand and there are no ambiguities (e.g. a gate which could be an OR or an AND, but it is not clear which one it is, label it to avoid confusion).
Each **solution** has to be justified and **the steps to get there have to be explicitly written down**, only providing the final outcome will lead to zero points. Always **include units** in your final answer where applicable. On **every page** please indicate which problem you are working on. If you separate the pages please indicate your name and student ID on **every page**.
For your convenience, you can find a page with basic equations related to the course material at the end of this document.

Full name: .... Student ID: ....

Problem 1 (20 points)

Consider the circuit given below with values $R_1 = 2\text{k}\Omega$, $R_2 = 0.5\text{k}\Omega$, $R_3 = 1\text{k}\Omega$, $R_4 = 1\text{k}\Omega$, $V = 18\text{V}$ and $I = 1\text{mA}$.



Throughout this problem, R_5 is taken to be the load resistor.

(a) (4 points) Compute the equivalent resistance R_{eq} as seen from R_5.

(b) (11 points) Using the variables given above, compute the open circuit voltage, V_{OC}, as seen from the load resistor R_5.

(c) (2 points) Select the true statements.

  (a) The equivalent resistance of an ideal current source is infinite.
  (b) The equivalent resistance of an ideal current source is zero.
  (c) The equivalent resistance of an ideal voltage source is infinite.
  (d) The equivalent resistance of an ideal voltage source is zero.

(d) (3 points) Draw the Norton and Thévenin equivalent circuits considering R_5 as a load resistor. Label the circuits with their names (Norton/Thévenin). Derive the expression for the current flowing through the load resistor R_5 using the Norton equivalent circuit.

Problem 1 - solution (20 points)

(a) (5 points) To compute the equivalent resistance seen by the load resistor R_5, current and voltage sources are opened and shorted respectively. This yields the diagram shown in Figure 1.

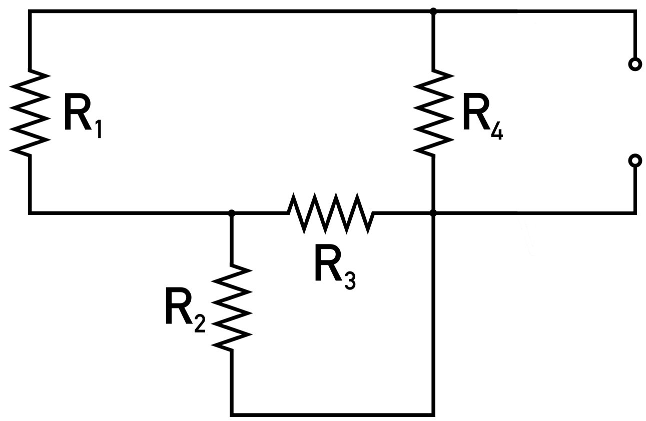


Figure 1: Equivalent resistance circuit.

Using the rules of parallel and series circuits, R_2 is in parallel with R_3, which in turn is in series with R_1. This is then in parallel with R_4, giving an expression:

$$R_{\text{eq}} = ((R_2//R_3) + R_1)//R_4 = \frac{\left[\frac{R_2 R_3}{R_2+R_3} + R_1\right] R_4}{\left[\frac{R_2 R_3}{R_2+R_3} + R_1\right] + R_4}$$

Since the question explicitly asks "to compute", a numerical value is expected. The given values in the problem can be substituted into the expression, giving a final value of $\underline{\underline{R_{\text{eq}} = 700\Omega}}$. Units must be used.

(b) (12 points) Usage of the superposition principle: Two cases need to be considered: the circuit with the voltage source shorted and the circuit with the current source open. Consider the latter first, shown in Figure 2.

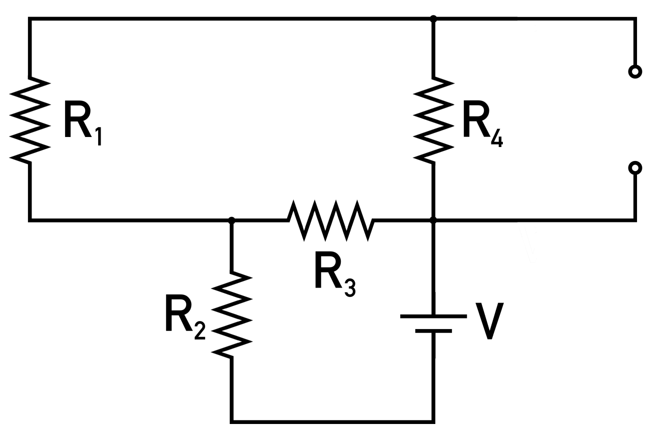


Figure 2: Circuit with the current source opened.

The voltage V' across the resistor R_4 is desired. To do this, the circuit can be simplified as shown in Figure 3.

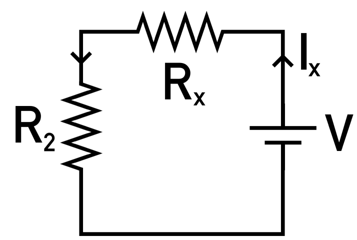


Figure 3: Simplification of circuit with current opened.

Here, resistors R_1 and R_4 are in series with one another which are in turn parallel to R_3, giving a net resistance R_x, equal to:

$$R_x = \frac{(R_1 + R_4)R_3}{(R_1 + R_4) + R_3}$$

This allows for the calculation of the current I_x, flowing along the same wire as the voltage source, given by:

$$I_x = \frac{V}{R_x + R_2}$$

With this current calculated, it must branch off into a current I_y flowing through the same wire as resistor R_4 in the original diagram, shown in Figure 4.

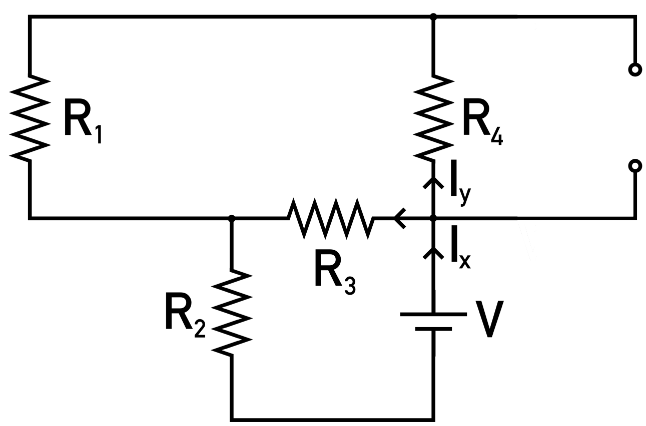


Figure 4: Open current circuit with the currents I_x and I_y illustrated.

The current I_y can be determined via usage of the current divider

$$I_y = I_x \frac{R_3}{R_3 + R_4 + R_1}$$

such that the voltage across R_4 and hence across the load resistor must be

$$V' = I_y R_4 = \frac{V}{\frac{(R_1+R_4)R_3}{(R_1+R_4)+R_3} + R_2} \cdot \frac{R_3}{R_3 + R_4 + R_1} \cdot R_4$$

which, when computed, gives $V' = 3.6$V. If we consider the node directly above R_4 as a reference point, it has a certain voltage, which must be less than the the voltage V at the node directly above the voltage source. We have used conventional current to define the direction of I_x, thereby resulting in a voltage gain with respect to the reference point defined above. So V' is positive.

Now, the voltage across R_4, and hence the voltage across the load resistor, needs to be found for when the voltage source is shorted. The circuit diagram of this situation is shown in Figure 5.

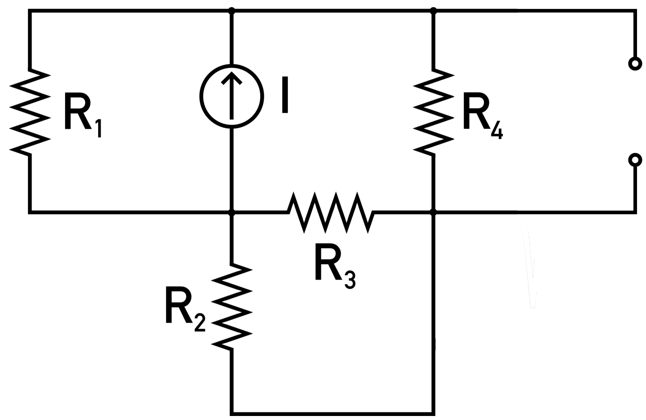


Figure 5: Circuit with the voltage source shorted.

It can be seen that R_2 is parallel to R_3, the net result denoted by R_a:

$$R_a = \frac{R_2 R_3}{R_2 + R_3}$$

From here, the current I must be split into two currents, denoted I_a and I_b, determined solely by the resistances, illustrated in Figure 6.

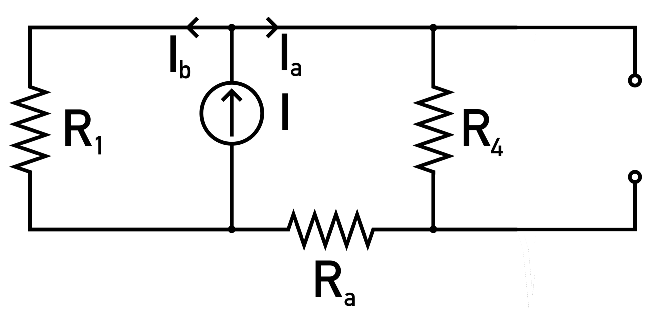


Figure 6: Circuit with the voltage source shorted, simplified with two currents I_a and I_b splitting from I.

The voltage V'', which is the same voltage as that across R_4, must correspond to

$$V'' = I_a R_4$$

where the current I_a can be found as:

$$I_a = I \frac{R_1}{R_1 + R_a + R_4} = I \frac{R_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4}$$

This final expression for the voltage V'' is then

$$V'' = I \frac{R_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4} \cdot R_4$$

Computing this voltage gives a value $V'' = 0.6\text{V}$. Considering the reference point as the node directly above R_4 as previously defined and keeping in mind that conventional current stipulates that a current source flows in the same direction as the current flowing from a voltage source (positive to negative terminal), the voltage at the reference point is greater than that of the node across the resistor, such that there is a voltage drop. Hence, $V'' = -0.6\text{V}$.

Using the superposition principle, $V_{\text{OC}} = V' + V''$, such that $\underline{\underline{V_{\text{OC}} = 3\text{V}}}$. Units must be used.

(c) (2 points) Statements (a) and (d) are correct.

(d) (3 points) The Norton and Thévenin equivalent circuits correspond to Figure 7. Using the

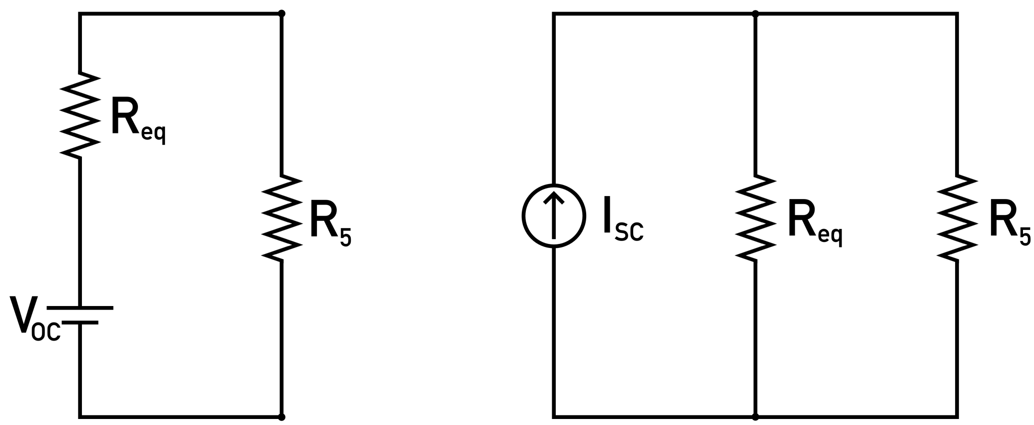


Figure 7: Thévenin (left) and Norton (right) equivalent circuits. $I_{\text{SC}} = V_{\text{OC}}/R_{eq}$.

diagram in Fig. 7(right), one can calculate the requested current I_{R_5} using the current divider equation:

$$I_{R_5} = I_{SC}\frac{R_{eq}}{R_5 + R_{eq}}$$

Alternatively, the voltage across R_5 can be first calculated:

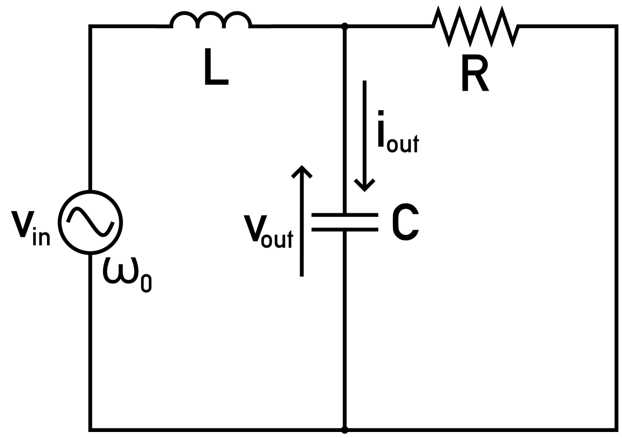
$$V = I_{SC}\left(R_5//R_{eq}\right) = I_{SC}\frac{R_5 R_{eq}}{R_5 + R_{eq}}$$

and then the current:

$$I_{R_5} = \frac{V}{R_5} = \frac{I_{SC}}{R_5}\frac{R_5 R_{eq}}{R_5 + R_{eq}} = I_{SC}\frac{R_{eq}}{R_5 + R_{eq}}$$

Problem 2 (16 points)

Consider the RLC circuit shown below. Assume sinusoidal regime, ideal components and the following parameters: $C = 0.0625\text{F}$, $L = 0.25\text{H}$, $R = 20\Omega$, $\omega_0 = 8\text{rads}^{-1}$.



(a) (3 points) Describe, without any calculations and only reasoning, the behaviour of

$$H(\omega) = \frac{i_{\text{out}}}{v_{\text{in}}}$$

for low ($\omega \to 0$) and high ($\omega \to \infty$) frequencies.

(b) (8 points) Using the parameter values given above, compute the value of the transfer function $H(\omega_0)$.

(c) (3 points) Suppose the resistor is a removable load. Determine the equivalent impedance of the circuit as seen from R as a function of the components in terms of the variables and find an expression of the kind:

$$Z_{\text{eq}} = jX$$

where X is purely real.

(d) (2 points) Select the true statements.

(a) Capacitors in parallel can be simplified using $C_{\text{eq}} = \sum C_i$.
(b) Capacitors in parallel can be simplified using $C_{\text{eq}}^{-1} = \sum C_i^{-1}$.
(c) Capacitors in series can be simplified using $C_{\text{eq}} = \sum C_i$.
(d) Inductors in parallel can be simplified using $L_{\text{eq}} = \sum L_i$.
(e) Inductors in series can be simplified using $L_{\text{eq}} = \sum L_i$.
(f) Inductors in parallel can be simplified using $L_{\text{eq}}^{-1} = \sum L_i^{-1}$.

Problem 2 - solution (16 points)

(a) (3 points) For low frequencies ($\omega \to 0$ rads^{-1}) the capacitor impedance $Z_C \to \infty$. Thus the capacitor may be replaced by an open circuit so no current flows through the capacitor and $i_{out} = 0$ and so $\underline{\underline{H(\omega) = 0}}$.

For high frequencies ($\omega \to \infty$ rads^{-1}) the inductor impedance $Z_L \to \infty$. Therefore, no current flows past the inductor and no current flows through the capacitor with $i_{out} = 0$. Therefore $\underline{\underline{H(\omega) = 0}}$.

(b) (8 points) The equivalent parallel impedance, Z_{RC} of the resistor and the capacitor must be found:

$$\frac{1}{Z_{\text{RC}}} = j\omega C + \frac{1}{R}$$
$$Z_{\text{RC}} = \frac{R}{j\omega RC + 1}$$

The voltage across this net impedance Z_{RC} corresponds to the voltage across the capacitor. The inductor now needs to be taken into consideration. The total impedance of the circuit is

$$Z_{\text{TOT}} = Z_{\text{RC}} + Z_{\text{L}}$$

and using the voltage divider:

$$v_{\text{out}} = \frac{Z_{\text{RC}}}{Z_{\text{TOT}}} v_{\text{in}} = \frac{Z_{\text{RC}}}{Z_{\text{RC}} + Z_{\text{L}}} v_{\text{in}} \implies v_{\text{out}} = \frac{R}{j\omega L + R(1 - \omega^2 LC)} v_{\text{in}}$$

Substituting the given parameter values in gives

$$v_{\text{out}} = -10 j v_{\text{in}}$$

which, when applying Ohm's law across the capacitor, yields i_{out}:

$$i_{\text{out}} Z_{\text{C}} = -10 j v_{\text{in}} \implies i_{\text{out}} = \frac{-10j}{-2j} v_{\text{in}} = 5 v_{\text{in}}$$

Re-arranging, the final computation of the transfer function $H(\omega_0)$ is

$$\underline{\underline{H(\omega_0) = 5\text{AV}^{-1}}} = 5\Omega^{-1}$$

where units **must** be given.

(c) (3 points) To find the equivalent impedance, the voltage source is shorted and the resistor is removed from the circuit, such that it is replaced by two open nodes. The capacitor is parallel to the inductor such that

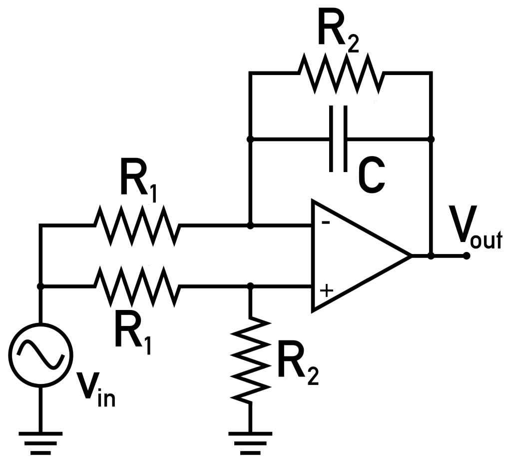
$$\frac{1}{Z_{\text{eq}}} = j\omega C + \frac{1}{j\omega L}$$
$$Z_{\text{eq}} = \frac{j\omega L}{1 - \omega^2 CL}$$

where $X = \omega L/(1 - \omega^2 CL)$.

(d) (2 points) Statements (a), (e) and (f) are correct.

Problem 3 (20 points)

Consider the following circuit with an ideal operational amplifier and an AC voltage input v_{in} at frequency ω.



(a) (2 points) State the two fundamental rules of ideal operational amplifiers **clearly**.

(b) (3 points) Find an expression for the voltage at the positive terminal of the operational amplifier v_+ in terms of v_{in} and the component impedances.

(c) (12 points) Hence, compute the transfer function defined by

$$H(\omega) = \frac{v_{\text{out}}}{v_{\text{in}}}$$

at frequency $\omega = \omega_0$ with $\omega_0 = 2/(R_2 C)$, $R_1 = 3\text{k}\Omega$ and $R_2 = 1\text{k}\Omega$.

(d) (3 points) Determine expressions for the transfer function at very large ($\omega \to \infty$) and very small ($\omega \to 0$) frequencies in terms of R_1 and R_2.

Problem 3 - solution (18 points)

(a) (2 points) 1. No current flows into the inputs and 2. When connected in a negative feedback configuration, the operational amplifier changes the output such that the two inputs are equal to one another.

(b) (4 points) The voltage at the positive terminal v_+ can be found using the potential divider:

$$v_+ = v_{\text{in}} \frac{R_2}{R_1 + R_2}$$

(c) (12 points) To solve this, it is important to properly analyze the circuit. To start, R_2 and C are in parallel with one another. The resultant impedance should be determined:

$$\frac{1}{Z_{||}} = \frac{1}{R_2} + j\omega C$$
$$Z_{||} = \frac{R_2}{j\omega R_2 C + 1}$$

Next, what should be recognized is that the current from the voltage source to v_{out}, flowing through R_1 and $Z_{||}$, is uniform. This means the expression

$$v_{\text{in}} - v_{\text{out}} = i(R_1 + Z_{||}) \tag{1}$$

is the key to determining the transfer function. The current i can be found considering the voltage across R_1. Per the fundamental rules of an ideal operational amplifier, $v_- = v_+$. The voltage across R_1 is simply $v_{\text{in}} - v_+$. This can be substituted into Ohm's law:

$$v_{\text{in}} - v_+ = iR_1$$
$$v_{\text{in}} - v_{\text{in}} \frac{R_2}{R_1 + R_2} = iR_1$$
$$i = \frac{v_{\text{in}}}{R_1 + R_2}$$

We now substitute the current into the expression relating v_{in} and v_{out}:

$$v_{\text{in}} - v_{\text{out}} = \frac{v_{\text{in}}}{R_1 + R_2}(R_1 + Z_{||})$$
$$v_{\text{out}} = \frac{v_{\text{in}}}{R_1 + R_2}(R_2 - Z_{||})$$
$$v_{\text{out}} = v_{\text{in}} \frac{R_2}{R_1 + R_2}\left(1 - \frac{1}{j\omega R_2 C + 1}\right)$$
$$v_{\text{out}} = v_{\text{in}} \frac{R_2}{R_1 + R_2} \frac{j\omega R_2 C}{j\omega R_2 C + 1}$$
$$v_{\text{out}} = v_{\text{in}} \frac{1}{R_1 + R_2} \frac{\omega^2 R_2^3 C^2 + j\omega R_2^2 C}{\omega^2 R_2^2 C^2 + 1}$$
$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{\omega^2 R_2^3 C^2}{(R_1 + R_2)(\omega^2 R_2^2 C^2 + 1)} + j \frac{j\omega R_2^2 C}{(R_1 + R_2)(\omega^2 R_2^2 C^2 + 1)}$$

The frequency ω_0 needs to be substituted such that

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{4R_2}{5(R_1 + R_2)} + j \frac{2R_2}{5(R_1 + R_2)}$$

and substituting R_1 and R_2 gives:

$$\underline{\underline{H(\omega_0) = 0.2 + 0.1j}}$$

(d) (4 points) If the frequency is very small, the impedance of the capacitor is infinite: $\mathrm{Z}_{||} = \infty$

From property OP-amp we know and applying voltage divider to to determine v_+;

$$v_+ = v_- = v_{in}\frac{R2}{R1+R2} \tag{2}$$

Kirchhoff's current law:

$$I_1 + I_2 = 0 \tag{3}$$

From Ohm's law the current through R1 and R2 can be determined:

$$I_1 = \frac{v_{in} - v_-}{R_1} \tag{4}$$

$$I_2 = \frac{v_{out} - v_-}{R_2} \tag{5}$$

Substituting Eq. 2, Eq. 4, and Eq. 5 into Eq. 3 gives;

$$\frac{1}{R_1}(v_{in} - \frac{R_2}{R_1+R_2}v_{in}) = \frac{1}{R_1+R_2}v_{in} - \frac{V_{out}}{R_2} \tag{6}$$

Rewriting gives;

$$\frac{V_{out}}{R_2} = \frac{1}{R_1}\frac{R_2}{R_1+R_2}v_{in} + \frac{v_{in}}{R_1+R_2} - \frac{v_{in}}{R_1} \tag{7}$$

$$\frac{v_{out}}{v_{in}} = \frac{1}{R_1}\frac{R_2^2}{R_1+R_2} + \frac{R_1}{R_1}\frac{R_2}{R_1+R_2} - \frac{R_2}{R_1}\frac{R_1+R_2}{R_1+R_2} \tag{8}$$

$$\frac{v_{out}}{v_{in}} = \frac{1}{R_1}\frac{R_2^2}{R_1+R_2} + \frac{1}{R_1}\frac{R_1R_2}{R_1+R_2} - \frac{1}{R_1}\frac{R_1R_2+R_2^2}{R_1+R_2} \tag{9}$$

$$\frac{v_{out}}{v_{in}} = \frac{1}{R_1}\frac{R_2^2 + R_1R_2 - R_1R_2 - R_2^2}{R_1+R_2} \tag{10}$$

$$\frac{v_{out}}{v_{in}} = 0 \tag{11}$$

Gives; $v_{out} = 0$

If the frequency is very large, the parallel impedance of the capacitor and the resistor R_2 needs to be considered

$$Z_{||} = \frac{R_2}{j\omega R_2 C + 1}$$

where, if $\omega \to \infty$, then the parallel impedance is equal to zero. Hence, the parallel network can be shorted, such that $v_+ = v_{\text{out}}$. In this case, the expression derived in part (b) becomes

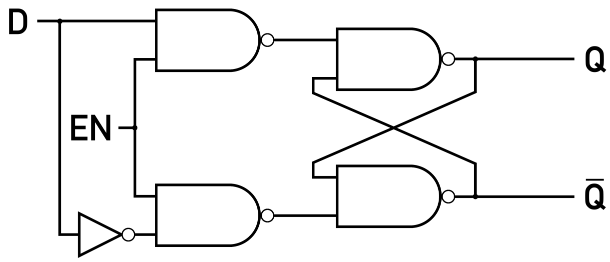
$$v_{\text{out}} = v_{\text{in}}\frac{R_2}{R_1+R_2}$$

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{R_2}{R_1+R_2}$$

such that $\underline{\underline{H(\omega \to \infty) = R_2/(R_1+R_2)}}$.

Problem 4 (14 points)

Shown below is the circuit of a D latch with outputs Q and $\overline{Q}$.

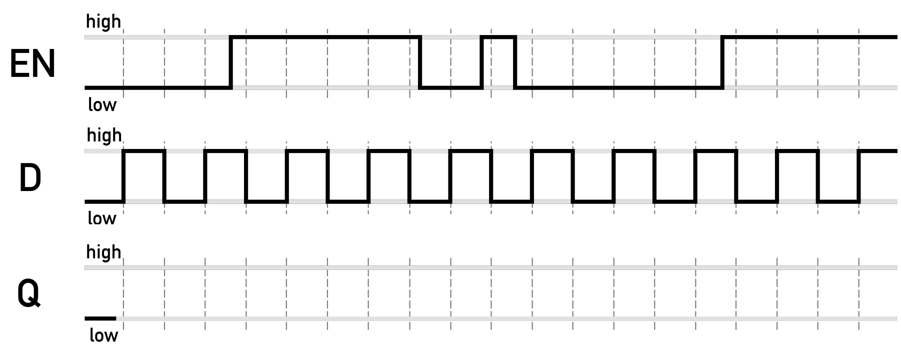


D	EN	Q_i	Q_{i+1}
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

(a) (4 points) Fill in the truth table above relating the inputs D and EN to the initial output Q_i and the next output Q_{i+1}.

(b) (2 points) Write the simplest boolean expression for $Q_{i+1} = F(D, EN, Q_i)$.

(c) (4 points) The diagram below shows the signal waveforms of the inputs EN and D. In response to these inputs, the output Q changes. Draw the signal waveform of the output Q on the diagram.



(d) (4 points) Suppose the inverter of a D latch circuit is removed (replaced by a short circuit). Fill in the truth table below for this case. For each line insert one of the following: 0, 1, IN (for invalid).

EN	D	Q_i	Q_{i+1}
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Problem 4 - solution (14 points)

(a) (4 points) By analyzing the circuit with the three given variables, the truth table below is obtained:

D	EN	Q_i	Q_{i+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(b) (2 points) Using either a Karnaugh map or by gathering the terms that produce an output of 1 in $Q_{\text{i+1}}$, the following expression is obtained:

$$Q_{i+1} = \overline{EN} \cdot Q_i + EN \cdot D$$

Solution I: Using a Karnaugh map for the variables EN, D and Q_i and filling it in from the truth table: By identifying the highlighted minterm groups, the simplified expression for $Q_{\text{i+1}}$

EN \ DQ_i	00	01	11	10
0	0	1	1	0
1	0	0	1	1

can immediately be obtained as:

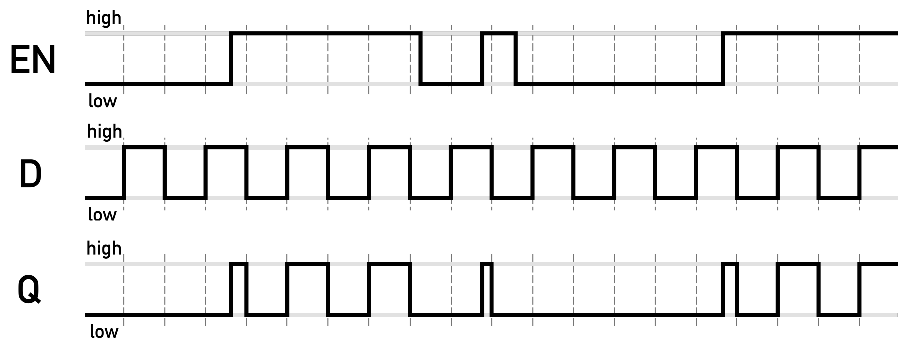
$$\underline{\underline{Q_{i+1} = \overline{EN} \cdot Q_i + EN \cdot D}}$$

Solution II: Using boolean algebra, the unsimplified boolean expression for $Q_{\text{i+1}}$ corresponds to

$$Q_{\text{i+1}} = \overline{EN} \cdot \overline{D} \cdot Q_i + \overline{EN} \cdot D \cdot Q_i + EN \cdot D \cdot \overline{Q_i} + EN \cdot D \cdot Q_i$$
$$Q_{\text{i+1}} = \overline{EN} \cdot (\overline{D} \cdot Q_i + D \cdot Q_i) + EN \cdot (D \cdot Q_i + D \cdot \overline{Q_i})$$

and since $\overline{A} + A = 1$, this can be further simplified to $\underline{\underline{Q_{i+1} = \overline{EN} \cdot Q_i + EN \cdot D}}$.

(c) (4 points) Using either the truth table or the boolean expression for Q_{i+1}, the signal for Q corresponds to:



(d) (4 points) Removing the inverter for the circuit changes truth table outputs where $EN = 1$. The resultant truth table corresponds to:

EN	D	Q_i	Q_{i+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	IN
1	1	1	IN

Fundamentals of Electronics - Formula Sheet

Ohm's law: $V = ZI$ Capacitors: $V = \frac{Q}{C} = \frac{1}{C}\int I \, \mathrm{d}t$ Inductors: $V = L\,\frac{\mathrm{d}I}{\mathrm{d}t}$

Complex impedance: $Z_R = R$, $Z_L = j\omega L$, $Z_C = \frac{1}{j\omega C}$ Reactance: $X = \mathfrak{Im}(Z)$

Quality ratio : $Q = \frac{f_0}{B}$ (for bandwidth B and resonance frequency f_0)

Root-mean-square voltage of sinusoidal signal = 0.707 of amplitude

Voltage gain: $A_v = \frac{V_o}{V_i}$ dB voltage gain = $20\log_{10}(\frac{V_o}{V_i})$

Closed loop gain: $G = \frac{A}{1+AB}$, where A is the forward gain and B is the feedback gain.

Output voltage of an OpAmp: $V_{out} = A\,(V_+ - V_-)$

Characteristic impedance of a cable: $Z_{eq} = \sqrt{\frac{r}{2\omega c}}(1-j)$ (RC cable), $Z_{eq} = \sqrt{\frac{l}{c}}$ (LC cable)

Speed of signal in a cable: $v = \frac{1}{\sqrt{lc}}$ (LC cable)

Effective impedance seen by a source connected to a cable (length Λ, Z_0) and a load with impedance Z: $Z_{eff} = Z_0\frac{Z - jZ_0 tan(k\Lambda)}{Z_0 - jZ tan(k\Lambda)}$, where $k = \frac{2\pi}{\lambda} = \omega\sqrt{lc}$

Boolean Algebra

- Commutative laws: $AB = BA$, $A + B = B + A$
- Distributive laws: $A(B+C) = AB + AC$, $A + BC = (A+B)(A+C)$
- Associative laws: $A(BC) = (AB)C$, $A + (B+C) = (A+B) + C$
- Absorption law: $A + AB = A\,(A+B) = A$
- De Morgan's laws: $\overline{A+B} = \overline{A}\cdot\overline{B}$, $\overline{AB} = \overline{A} + \overline{B}$
- Other: $A + \overline{A}\cdot B = A + B$, $A\,(\overline{A} + B) = AB$

Complex Numbers Algebra

$$|z| = \sqrt{\mathfrak{Re}(z)^2 + \mathfrak{Im}(z)^2}$$

$$e^{j\phi} = \cos(\phi) + j\sin(\phi) \implies e^{j\frac{\pi}{2}} = j\ ,\ e^{-j\frac{\pi}{2}} = -j\ ,\ e^{j\pi} = -1 = j^2$$

$$\cos(\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2} \qquad \sin(\theta) = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$